APL SIMD Boolean Array Algorithms

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October 5, 2016

Abstract

Computation on large Boolean arrays is becoming more prevalent, due to applications such as cryptography, data compression, and image analysis and synthesis. The advent of bit-oriented vector extensions for microprocessors and of GPUS presents opportunities for significant performance improvements in such Boolean-dominated applications. Since APL is one of the few computer languages that supports dense (one bit per element, eight bits per byte), multi-dimensional Boolean arrays as first-class objects, it has naturally attracted research into optimizations for improved performance of Boolean array operations. This paper presents some of the Single Instruction, Multiple Data (SIMD) Boolean-related optimizations that have appeared in APL implementations, and suggests ways in which those optimizations might be exploited using contemporary hardware.

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- Boolean arrays are grist to APL's data-parallel, expressive mill!

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- GPU and SIMD vector facilities can exploit them

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- without doing *any* element-wise computations

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- rbemove will copy 12 adjacent array elements at once

Structural and Selection Verbs III

- 2 3 4⍴⍳24

```
 0  1  2  3
 4  5  6  7
 8  9 10 11

12 13 14 15
16 17 18 19
20 21 22 23
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- then byte-aligned the resulting vector, SIMD, a word at a time

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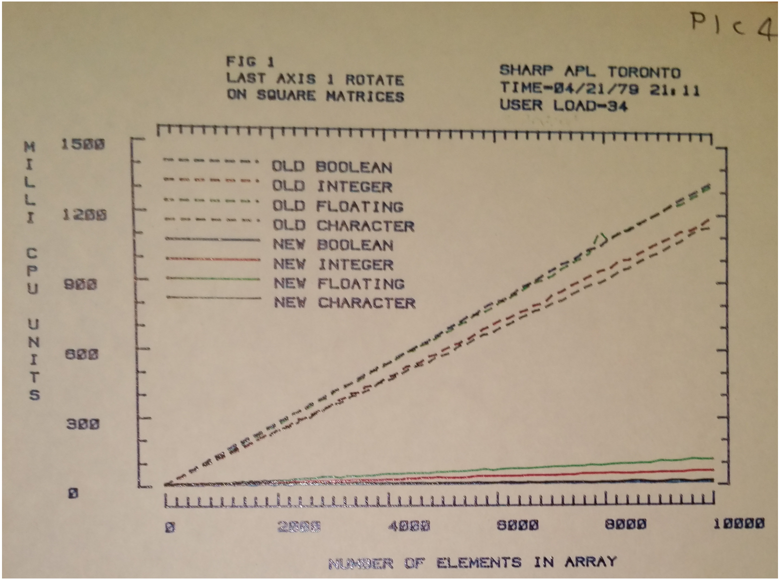
- All non-last-axis operations copied entire cells at once, using rbemove

Reverse and Rotate Performance on Booleans



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- Do an overlapped move, or “smear" of the result to its tail

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- Kernel generalizes to any power of two, *e.g.*, 16x16, 32x32

Search Verbs

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- Created indexof kernel utility for interpreter use

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- Algorithm used briefly for ∨/ω and ∧/ω

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- Result was catenation of prefix, infix, & suffix expressions
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- Result: linear-time, word-at-a-time, SIMD Boolean scan & reduce

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  biw←⌈2⍟1⌈⍴y
  :For s :In 2*⍳biw ⍝ Heckman
    r←r≠(-⍴r)↑(-s)↓r
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- Bernecky's simple C Heckman implementation is about 3X faster than Dyalog APL 15.0 (vector only)

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r←nescanall y;s;biw
  ⍝ Not-equal scan
  r←y
  biw←⌈2⍟1⌈⍴y
  :For s :In 2*⍳biw ⍝ Heckman
    r←r≠(-⍴r)↑(-s)↓r
  :EndFor
```

- SIMD, word-at-a-time algorithm for Boolean ≠\⍵ and =\⍵ along last axis
- Bernecky's simple C Heckman implementation is about 3X faster than Dyalog APL 15.0 (vector only)
- So far, no X86 vectorization; perhaps we can do even better

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- redesigned Boolean inner product to use STAR algorithm

Classic Inner Product Algorithm

```
Z←X ipclassic Y;RX;CX;CY;I;J;K
 RX←(⍴X)[0]
 CX←(⍴X)[1]
 CY←(⍴Y)[1]
 Z←(RX,CY)⍴0.5
 :For I :In ⍳RX
  :For J :In ⍳CY
   Z[I;J]←0
   :For K :In ⍳CX
    Z[I;J]←Z[I;J]+X[I;K]×Y[K;J]
   :EndFor
  :EndFor
 :EndFor
```

STAR Inner Product Algorithm

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Z←X ipstar Y;RX;CX;CY;I;J;Xel
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 CY←(⍴Y)[1]
 Z←(RX,CY)⍴0
 :For I :In ⍳RX
  :For J :In ⍳CX
   Xel←X[I;J]
   Z[I;]←Z[I;]+Xel×Y[J;]
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  `Z[I;]←Z[I;] f tmp`

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- Unfortunately, the APL primitive is still 30X faster than the APL model

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- This gives us poor man's sparse arrays, which works on other data types, too
- Final result: Boolean inner products on SHARP APL/PC ran much faster than APL2 on huge mainframe

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- See also the Bernecky-Scholz PLDI2014 Arrays Workshop paper: *Abstract Expressionism*

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- Second phase can be performed in SIMD, *e.g.*, by a single SAC data-parallel with-loop.

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- SIMD Boolean upgrade:
  `ug←{((~ω)/⍳⍴ω),ω/⍳⍴ω}`

Boolean Grade

- SIMD Boolean upgrade:
  `ug←{((~⍵)/⍳⍴⍵),⍵/⍳⍴⍵}`
- Not stunningly SIMD, though.

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- A similar definition holds for word-oriented algorithms

Acknowledgements

My sincere thanks to James A. Brown, Larry M. Breed, Walter Fil, Jay Foad, Roger K.W. Hui, Roger D. Moore, and Bob Smith, for their constructive suggestions and ideas regarding this paper.

The APL programs in this paper were executed on Dyalog APL Version 15.0. Dyalog provides free downloads of their interpreters for educational use; they also offer a free download of their Raspberry Pi edition, at `www.dyalog.com`.

The British APL Association (BAA) provided the author with financial assistance for attending this conference; their thoughtfulness and generosity is greatly appreciated.